# Static and dynamic length scales in glass forming liquids Paddy Royall

"The perceived wisdom is that structure determines dynamics" - Peter Harrowell



Royall and coworkers,

"Complex plasmas and colloidal dispersions: particle-resolved studies of classical liquids and solids", World Scientific (2012)"

# Acknowledgements

Tannie Liverpool

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# Royall group, Bristol, England

Ryoichi Yamamoto (Kyoto)

Thomas Speck (Mainz)

#### Why do we expect structure to play a role in the glass transition?

#### How do we measure - and identify - the relevant structure?

#### Is structure really a cause for slow dynamics?

- coincidence of structural and dynamic length scales
- structural correlations in the isoconfigurational ensemble
- vitrification by changing structure the  $\mu\text{-ensemble}$

# **The Angell plot**

#### **Royall/Structure**

lines are VFT fits

Fragility->more than one form of relaxation

Well described by Vogel-Fulcher-Tamman (VFT)

$$\tau_{\alpha} = \tau_0 \exp\left(\frac{A}{T - T_0}\right)$$

Hard spheres : equivalent to T is reduced pressure Berthier and Witten PRE **80** 021502 (2009)

$$\tau_{\alpha} = \tau_0 \exp\left(\frac{A}{Z_0 - Z}\right)$$

$$Z = \frac{\Pi}{k_B T \rho}$$

Pressure from Carnahan-Starling EoS

$$\Pi = nk_BT \frac{1+\phi+\phi^2-\phi^3}{(1-\phi)^3}$$

#### HS hard sphere colloids



Richert and Angell JCP **108**, 9016 (1998) inspired by Angell J. Non-Cryst. Solids **102**, 205–221 (1988)

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## **Cooperatively rearranging regions Adam-Gibbs and RFOT**

Assume a group of molecules which relax and leave the others fixed

Adam-Gibbs theory assumes a few (M) states accessible to the molecules in the cavity of size  $\xi^3$ 

$$S_{\rm conf}(T) \sim k_{\rm B} \xi^{-3} \ln M$$

Assume energy barrier to re-arrangement ~  $\xi^3$  The time to rearrange between these M states is ~

$$\tau_{\alpha} = \tau_0 \exp\left[\frac{C_0 \ln M}{TS_{\rm conf}(T)}\right] \, {\rm VFT}$$

Adam and Gibbs JCP 43, 139-146 (1965)

Random First Order Theory

A first-order transition to a random mosiac state

Like crystallisation but the low-T state has very many configurations

Relaxation via entropic nucleation.  $TS_{conf}(T)\xi^3$ 

Relaxation opposed by surface tension  $\Upsilon \xi^{ heta}$ 

Equate for mosiac lengthscale 
$$\xi = \left(\frac{\Upsilon}{Ts_c(T)}\right)^{1/(3-\theta)} \quad \log\left(\frac{\tau_{\alpha}}{\tau_0}\right) = c \frac{\Upsilon}{k_B T} \left(\frac{\Upsilon}{Ts_c(T)}\right)^{\psi/(3-\theta)}$$

Lubchenko and Wolynes Ann. Rev. Phys. Chem. 58, 235-66 (2007)



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- Both Adam-Gibbs and RFOT suggest a growing lengthscale upon supercooling
- Montanari-Semmerjian : at sufficient cooling, there must be a growing lengthscale for super-Arrhenius dynamics



So we would expect a growing structural lengthscale ...but what is the structure?

"The arrangement of atoms and molecules in glass is indistinguishable from that of a liquid."



Neutron scattering on propylene glycol ~ Leheny et. al. J. Chem. Phys. 1996

# Why have we not got a crystal?



Sir Charles Frank Physics 1946-1998



#### Supercooling of liquids

By F. C. Frank

H. H. Wills Physics Laboratory, Bristol University

The theoretical argument is misleading also. Consider the question: 'In how many different ways can one put twelve billiard balls in simultaneous contact with one, counting as different the arrangements which cannot be transformed into each other without breaking contact with the centre ball?' The answer is three. Two which come to the mind of any crystallographer occur in the face-centred cubic and hexagonal close-packed lattices. The third comes to the mind of any good schoolboy, and is to put one at the centre of each face of a regular dodecahedron. That body has <u>five-fold</u> axes, which are abhorrent to crystal symmetry: unlike the other two packings, this one cannot be continuously extended in three dimensions. You will find that the outer twelve in this packing do not touch each other. If we have mutually attracting deformable spheres, like atoms, they will be a little closer to the centre in this third type of packing; and if one assumes they are argon atoms (interacting in pairs with attractive and repulsive energy terms proportional to  $r^{-6}$  and  $r^{-12}$ ) one may calculate that the binding energy of the group of thirteen is 8.4% greater than for the other two packings. This is 40% of the lattice energy per atom in the crystal. I infer that this will be a very common grouping in liquids, that most of the groups of twelve atoms around one will be in this form, that freezing involves a substantial rearrangement, and not merely an extension of the same kind of order from short distances to long ones; a rearrangement which is quite costly of energy in small localities, and only becomes economical when extended over a considerable volume, because unlike the other packing it can be so extended without discontinuities.

#### Frank, Proc. R. Soc. 215 43 (1952)

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#### **Geometric frustration**

In some non-frustrated scenario, there is a continuous transition to an "ideal glass" of the locally favoured structure (LFS) of the liquid.

120 spheres tesselate into icosahedra on the surface of a 4D hypersphere

...back in the real world...

The growth of domains of LFS are frustrated. Free energy :

$$F(\xi, T) = \Upsilon(T)\xi^{\theta} + \delta F_{\text{bulk}}(T)\xi^{3}$$

classical nucleation theory

 $\boldsymbol{\xi}$  measure of the LFS domain size

Is curved space vs Euclidean space the only frustration scenario?

curved 3D space on 4D hypersphere forms an "ideal glass" of 120 identical spheres but we know identical spheres in 3D are not an ideal glassformer

 $\delta F_{BULK}\,$  change in bulk free energy between "crystal" and liquid





#### **Royall/Structure**

Tarjus et al. J. Phys: Condens. Matter 17, R1143 (2005)

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frustration

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Structure and glass : beyond the icosahedron



how to identify structures in bulk systems



## 5-membered ring cluster

How to identify five-membered rings in bulk?



how to identify structures in bulk systems



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Clusters can overlap

how to identify structures in bulk systems



#### 5-membered ring cluster

How to identify five-membered rings in bulk?



Strategy:

Search for clusters in bulk, for m<14. If small clusters contained within larger, only consider larger Also identify BCC, FCC and HCP





**Experimental "hard sphere" data** 

# **Dynamic Toplogical Cluster Classification**

#### Linking structure and dynamics



## But what about Lennard-Jones...and Frank's Icosahedra?

Binary Lennard-Jones mixture (Wahnstrom) additive,  $\sigma_A=5/6\sigma_B$ . Molecular Dynamics simulation



# Icosahedra domain growth upon cooling

**Royall/Structure** 

Binary Lennard-Jones mixture (Wahnstrom) additive,  $\sigma_A=5/6\sigma_B$ . Molecular Dynamics simulation



#### Emergence of network of icosahedral (slow) particles

# **Dynamic Toplogical cluster classification**

**Royall/Structure** 

Linking structure and dynamics

Kob-Andersen (80:20), non-additive,  $\sigma_{AA}=1 \sigma_{BB}=0.88$ 



11A lasts much longer than all other clusters

# **Change in dynamics...and structure**

**Royall/Structure** 

Kob-Andersen (80:20), non-additive,  $\sigma_{AA}=1 \sigma_{BB}=0.88$ 



# **11A domain growth upon cooling**

**Royall/Structure** 

#### Kob-Andersen (80:20), non-additive, $\sigma_{AA}=1 \sigma_{BB}=0.88$



#### Emergence of network of particles in 11A clusters

# **Experiments!**

## "Hard" spheres - "quench" by increasing density



effective colloid volume fraction  $\varphi$ 

# **Dynamic TCC - cluster lifetimes**

Hard spheres (MD)



Royall et al. proceedings of this meeting

## **Change in structure in hard spheres**



Similar to Lennard-Jones models 10B increases with compression. Falling out of equilibrium  $(\phi=0.585)$ : 5A triangular bipyramid

Royall et al. proceedings of this meeting



# Experimental data at $\phi$ =0.585. Network of 10B

#### **Royall/Structure**

#### **Lennard-Jones models**

Wahnstrom (50:50), additive,  $\sigma_{AA}=1 \sigma_{BB}=0.833$ icosahedron (13A) - Coslovich 2007 ...and Frank-Kasper bonds - Pedersen 2010 Royall and coworkers *JCP* **138** 12A535 (2013)

Kob-Andersen (80:20), non-additive, отал=1 отве=0.88 bicapped square anti-prism (11А) - Coslovich 2007 Malins, Eggers, Tanaka and Royall *Faraday Disc.* **167** paper 16 (2013)

## **Colloid experiments**

Particle-resolved studies of colloids `Hard' spheres (+ MD simulations) 6-8% polydisperse Royall *et al. proceedings of this meeting* 



#### Each model has its own locally favoured structure

#### **Royall/Structure**

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Malins, Eggers, Tanaka and Royall Faraday Disc. 167 paper 16 (2013)

30

Fraction of full icosahedra (%)

5

0

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## **Generality???**

Icosahedra in embedded atom model simulations of CuZr Cheng, Sheng and Ma PRB 78, 014207 (2008)

#### 11A bicapped square anti-prisms in Al-based alloys

Evteev et al. Acta. Mater. 51 2665 (2003)

cf Ken Kelton's talk



## **Change in structure in different systems**

**Royall/Structure** 



correlation with fragility ??

Royall et al. proceedings of this meeting

#### **Geometric Frustration**

$$F(\xi, T) = \Upsilon(T)\xi^{\theta} + \delta F_{\text{bulk}}(T)\xi^3 + s(T)\xi^5$$

classical nucleation theory

frustration





Tarjus *et al. J. Phys: Condens. Matter* **17**, R1143 (2005)

*T*=0.620 We see a lot of networks of locally favoured structures

#### 1D length compatible with strong frustration

strong frustration : Charbonneau<sup>2</sup>, Tarjus JCP **138** 12A515 (2013)

# Investigating isomorphs with the TCC

**Royall/Structure** 



Kob-Andersen (80:20), non-additive,  $\sigma_{AA}=1 \sigma_{BB}=0.88$ 

Inverse power law (IPL) mapped to Lennard-Jones following isomorphism

Dyre *PRE* **87** 022106 (2013)

2-point similar to Pedersen *et al. PRL* **105** 157801 (2011), 2-point dynamics agree



Malins, Eggers, Royall JCP 139 234505 2013 (2013)

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increase in 11A lifetime in LJ

Malins, Eggers, Royall JCP 139 234505 2013 (2013)



if structure were a cause for the glass transition we might expect structural lengthscales to grow with dynamic lengthscales

Do the lengthscales grow together?

Yes! Tanaka Nature Materials (2010), Nature Comms (2012), Mosayebi et al PRL (2010) and more... Non! Famille Charboneau and Tarjus PRL (2011), Karmakar et al PNAS (2009) Kob et al. Nature Physics (2011), Charbonneau and Tarjus JCP (2013), Hocky et al PRL (2012), Dunleavy et al. PRE (2012) and more...

# **Dynamic lengthscales**

#### Wahnstrom Binary Lennard-Jones



 $< r^{2}(t_{h}) >: \circ < 0.043$ 

So far - structure and local influence

What are the dynamics (and how do they couple to the structure)

Spatially heterogeneous dynamics

Cool supercooled liquid - increasing dynamic correlation length  $\xi_4$  - lengthscale of dynamically heterogeneous regions

 $\xi_4$  - "standard definition" - fit low q end of SSIowSIow(q) = 1/(1- $\xi_4^2$  q<sup>2</sup>)

Lacevic *et al. JCP* **119** 7372 (2003)

Also have structural correlation lengths.

 $\xi$ S13A "standard definition" for icosahedra

 $\xi_{\text{Rg}}$  radius of gyration of domains of icosahedra

Malins, Eggers, Royall, Williams and Tanaka JCP 138 12A535 (2013)





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see also Famille Charboneau and Tarjus PRL (2011), Tanaka Nature Mat (2010), Kob et al. Nature Physics (2011)

# Dynamic and static lengthscales do not scale together



Dynamic lengthscales and static lengthscales do not scale together

Malins, Eggers, Tanaka and Royall *Faraday Disc.* **167** paper 16 (2013) Malins, Eggers, Royall, Williams and Tanaka *JCP* **138** 12A535 (2013)

# What happens to the dynamic lengthscale ????





we are used to cooling/compressing a system for solidification

# A glass transition without cooling



*s*=0 no biasing (normal simulation)

#### The s-ensemble

Trajectory space sampling at T>glass transition (T=0.6)

Mobility c of trajectory of ~216 particles

Apply field *s* such that trajectories with low mobility (*c*) are selected

Hedges, Jack, Garrahan and Chandler Science **323** 1309 (2009)

Kob-Andersen binary Lennard Jones

Speck Malins and Royall *PRL* **109** 195703 (2012)

# A glass transition without cooling



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Hedges, Jack, Garrahan and Chandler Science **323** 1309 (2009)

Evidence for first-order transition

Speck Malins and Royall PRL 109 195703 (2012)

# A glass transition by biasing structure??

**Royall/Structure** 



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Trajectory space sampling at T>glass transition (T=0.6)

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Hedges, Jack, Garrahan and Chandler Science **323** 1309 (2009)

What about structure?

Jack, Hedges, Garrahan and Chandler PRL **107**, 275702 (2011) :

Very stable states from *s*-ensemble - have these a different structure??

Kob-Andersen -> increase in 11A?

Structure as the biasing field?

The  $\mu$ -ensemble



*K* length of trajectory

# A glass transition by biasing structure??

**Royall/Structure** 



# **Unified dynamical and structural transition**

**Royall/Structure** 



## $\mu\text{-ensemble corresponds to exceptionally deep quench$



TVFT T at which structural relaxation time diverges according Vogel-Fulcher-Tamman law

# **The Angell plot**



#### Thanks for your attention



Out now!

## **Our soft matter workshop**





Andrew

Rob Nigel Dave Karoline Dunleavy C Jack Claudia Wilding Philips Wiesner Tom Clem Woolston Phil Forreiro Phil Vicente Chris Peter lan Sánchez Fullerton Thompson Rhys Harrowell Jean-Francois Jade Taffs Fenech Law lan Wheater Camenen Zhang Bassindale Williams

## **Our soft matter workshop**



#### Protocol for our meeting



Theorists must know the acronym PMMA

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Soft matter experimentalists must be able to describe the physical basis of this plot

# Static and dynamic length scales in glass forming liquids

Topological cluster classification - a zoo of locally favoured structures

Locally favoured structures - modelspecific

Strong frustration - little linear growth network of locally favoured structures

Decoupling between ξ4 and ξstruct in the accessible regime. Deeper quenching???

Isoconfigurational ensemble : local structure for high mobility and a solution to the discrepancy in ξ4 and ξstruct?

Two large deviation ensembles - s and  $\mu$ . Both concern the same transition.

µ-ensemble melting : structure->slow dynamics

Wahnstrom : *JCP* **138** 12A535 (2013) KA : *Faraday Disc.* **167** paper 16 (2013) Hard spheres and frustration : proceedings of this meeting

